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**THREE-DIMENSIONAL ELASTIC FINITE ELEMENT
ANALYSIS OF NOTCHED TENSION SPECIMENS**

Report No. 87-220-K

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Nondestructive and Fracture Evaluation Section
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ABSTRACT

Finite element analyses were performed on idealizations of two types of notched tension specimens used in delayed hydride crack initiation experiments performed on Zr-2.5% Nb pressure tube material. Linear elastic plane stress, plane strain and three-dimensional analyses were performed to obtain tensile and von Mises stress values in the specimens. The above analyses yielded average stress concentration factors of 7.93 and 4.47 for the sharp notched and blunt notched specimens, respectively. Close agreement was obtained with stress concentration factors calculated from closed-form solutions. Plane strain constraint evaluations indicate plane strain conditions exist near the notch root in the sharp notch specimen. Near the root in the blunt notch specimen, the stress field is two-dimensional in nature and lies between plane stress and plane strain.

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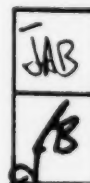
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EXECUTIVE SUMMARY

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OF NOTCHED TENSION SPECIMENS**

**F. Kwok and D.A. Scarth
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In support of the experimental efforts of the Materials Integrity Section of the Metallurgical Research Department on delayed hydride crack initiation, finite element analyses were performed on idealizations of two types of notched tension specimens used in the experiments. Linear elastic plane stress, plane strain and three-dimensional analyses were performed to obtain tensile and von Mises stress values in the specimens. Results were compared with classical methods involving the linear-elastic fracture mechanics stress intensity factor-based and stress concentration factor-based solutions. The degree of plane strain constraint was also evaluated at the notch tip for the three-dimensional models. The above analyses yielded average stress concentration factors of 7.93 and 4.47 for the sharp notched and blunt notched specimens, respectively. Plane strain constraint evaluations indicate plane strain conditions exist near the notch root in the sharp notch specimen. Near the root in the blunt notch specimen, the stress field is two-dimensional in nature and lies between plane stress and plane strain.



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To

Dr. D. Mills
Director of Research

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1.0 INTRODUCTION

Experiments are currently underway in the Materials Integrity Section of the Metallurgical Research Department on delayed hydride crack initiation in notched tension specimens fabricated from Zr-2.5% Nb pressure tube material. In order to gain an understanding of the stress distributions in these notched specimens, linear elastic finite element studies were performed on two notched tension specimens having different notch root radii. This report contains a summary of the analysis procedure used and the results obtained.

2.0 ANALYSIS PROCEDURE

2.1 Specimen Geometry

Idealized models of the notched tension specimens were adopted for the finite element study, as illustrated in Figures 1 and 2. The specimens have a thickness b , a width $2w$, and contain a notch of depth a and root radius ρ . To negate the edge effects in the direction of load application, the models were given an elongated length three times the wall thickness on either side of the notch.

2.2 Material Properties

Isotropic properties were used. Poisson's ratio, ν , was 0.43.

2.3 Loading

A uniform tensile stress was applied to the ends of the specimens.

2.4 Finite Element Analysis

The finite element computer code ABAQUS Version 4.6 /1/ was used to perform the finite element analyses.

2.4.1 Two-Dimensional Finite Element Analysis

Due to symmetry, only one-half of the specimen was modelled. For the sharp notch specimen, a mesh of 1007 nodes and 308 elements was adopted (Figure 3). Two meshes were used for the blunt notch specimen in order to facilitate a convergence study. The finer mesh was comprised of 1802 nodes and 563 elements, whereas the coarser mesh had 764 nodes and 231 elements (Figures 4 and 5). For each of the three meshes, plane stress and plane strain analyses were performed with the ABAQUS CPS8 and CPE8 elements, respectively.

For the blunt notch specimen, results obtained from the coarse and fine meshes are identical for all practical purposes. As a result, it was assumed that the coarse mesh is adequate, and was used for the remainder of the study.

2.4.2 Three-Dimensional Finite Element Analysis

A three-dimensional finite element mesh that models a quarter section of the tension specimen was generated for each of the two specimens. The meshes were generated by projecting the two-dimensional meshes (the coarse mesh for the blunt notch model) into the through-width (z) direction to form three element layers. The sharp notch model consists of 5078 nodes and 924 elements, and is shown in Figure 6. The blunt notch model consists of 3857 nodes and 693 elements, and is shown in Figure 7. Three-dimensional analyses were performed by using the ABAQUS C3D20R 20-node reduced-integration solid element.

2.5 Two-Dimensional Classical Stress Analysis

2.5.1 Stress Distribution Based on the Linear Elastic Fracture Mechanics Stress Intensity Factor

An approximate solution based on the stress intensity factor (K_I) associated with linear elastic fracture mechanics is available from reference /2/ to calculate the local stress distribution around a sharp notch. The tensile stress solution along the y-axis (refer to Figure 1) was superimposed with the far field solution, and is given by:

$$\sigma_{xx} = \frac{K_I}{(2\pi r)^{1/2}} + \frac{K_I}{(2\pi r)^{1/2}} \frac{\rho}{2r} + \sigma \quad (1)$$

where σ_{xx} = normal stress in the x-direction,
 σ = remote applied stress,
 K_I = stress intensity factor,
 ρ = radius of the notch root (see Figure 1),
 r = radial distance from the origin, centered a distance $\rho/2$ behind the notch tip.

The stress intensity factor K_I is calculated from:

$$K_I = \sigma (\pi a)^{1/2} F(a/b) \quad (2)$$

where a is the notch depth and b is the specimen thickness.

From page 2.11 of reference /2/, the geometry correction factor F is 1.50.

2.5.2 Stress Concentration Factors Based on Stress Intensity Factor

The stress concentration factor is the normal stress σ_{xx} at the notch root divided by the remote stress σ . The stress concentration factor at the root of the notch predicted by the stress intensity factor approach is obtained by substituting equation (2) into (1), dividing by the remote stress σ , and evaluating the result at $r = \rho/2$.

$$k_t = 2 \left(\frac{a}{\rho} \right)^{1/2} \cdot F(a/b) + 1 \quad (3)$$

2.5.3 Conventional Stress Concentration Factors

The centre of application of the axial remote stress is offset from the centroid of the net section at the notch. A bending moment is therefore induced across the reduced section. Stress concentration factors for a bar containing a U-notch in axial tension and in-plane bending are available from equations (3a) and (3b) of Table 37 in /3/. The same values can also be obtained from Figures 22a and 39a of reference /4/. A modified relation that combines both tensile and bending stress concentration effects due to the uniform axial stress on the unnotched cross-section is given as:

$$k_t = \frac{1}{(1 - a/b)} \left[k_a + \frac{3 k_b}{(b/a - 1)} \right] \quad (4)$$

where k_a and k_b are the stress concentration factors due to tension and bending applied to the net section, respectively. This relation produces the stress concentration factor based on the remote stress, k_t .

2.5.4 Plane Strain Constraint Factor

A plane strain constraint factor, γ , is defined as:

$$\gamma = \frac{\sigma_{zz}}{\sigma_{xx} + \sigma_{yy}} \quad (5)$$

where σ_{xx} = normal stress in the x direction,
 σ_{yy} = normal stress in the y direction,
 σ_{zz} = normal stress in the z direction.

The plane strain constraint factor was evaluated along the front of the notch root for the three-dimensional finite element models.

3.0 RESULTS AND DISCUSSION

3.1 Sharp Notch Specimen

3.1.1 Comparison of Stress Concentration Factors of Sharp Notch

Table 1 shows the stress concentration factors of the sharp notch specimen as calculated by the different approaches. Also included are the percentage differences which are defined as:

$$\% \text{ difference} = \frac{\text{finite element result} - \text{2D classical result}}{\text{2D classical result}} \times 100 \quad (6)$$

The difference between the finite element plane stress and plane strain solutions is due to numerical approximations, and should be zero. The maximum difference between the finite element and classical results amounts to a percentage difference of 6%.

TABLE 1
Stress Concentration Factors and Percentage Differences
for Sharp Notch Specimen

Sharp Notch	Stress Intensity Factor Solution (Eq 3)		Stress Concentration Factor Solution (Eq 4)	
	k	% Diff.	k	% Diff.
2D Classical Solution	7.71	0.00	7.75	0.00
FE Solution Plane Stress	7.90	2.46	7.90	2.00
FE Solution Plane Strain	8.11	5.12	8.11	4.65
FE Solution 3D, Centre of Specimen	8.20	6.36	8.20	5.81

3.1.2 Stress Distribution of Sharp Notch Specimen

Figure 8 shows the normalized tensile stress distribution in the through-thickness direction of the sharp notch specimen as obtained by the finite element and theoretical stress intensity factor based approach (Eq 1). For the sharp notch specimen, the origin $r = 0$ is located at a distance of $\rho/2$ behind the notch root (see Figure 1). Normalization was performed by dividing the stress values by the applied stress, σ . The finite element results were obtained from the two-

dimensional plane stress analysis, which yielded tensile stress values which are the same as the plane strain values. The theoretical approach is originally derived for the stress field in the close vicinity of a crack tip, and is adjusted to approximate the stress distribution around a relatively sharp notch. Due to this underlying assumption, good agreement between the two solutions can only be found in the notch tip region. It is believed that the finite element results would give a more accurate stress distribution. This approach was not applied to the blunt notch, as the root radius of the blunt notch is larger than the valid limit for this approximation.

Figure 9 shows the finite element results for the tensile stress distributions for the sharp notch specimen through the thickness of the specimen. The two-dimensional solution is very close to the centre plane solution of the three-dimensional model ($z/w=0$). This suggests the existence of stress conditions in the centre of the specimen that resembles the two-dimensional idealizations. This is further supported by the plane strain constraint factor calculations, as explained later in this report.

One measure of the propensity for plastic deformation is the von Mises equivalent stress, $\bar{\sigma}$, which is defined as:

$$\bar{\sigma} = \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (7)$$

where σ_1 , σ_2 and σ_3 are the principal stresses.

The two and three-dimensional von Mises equivalent through-thickness stress distributions for the sharp notch are given in Figure 10. Due to the additional stress component in the out of plane direction (σ_{zz}), the plane strain von Mises stress values are lower than the plane stress values. The values obtained at different planes through the width (z/w 's) from the three-dimensional model are generally bounded by the plane stress and plane strain solutions. An exception is near the notch root, where the von Mises stress for the three-dimensional model at $z/w = 1$ is the lower bound, due to the low value of σ_{xx} .

Figure 11 illustrates the through-thickness tensile stress distribution at different through-width planes in the three-dimensional model. The tensile stress values show an almost constant profile in z from $z/w = 0$ to $z/w = 0.5$, which is followed by an abrupt drop in values that reach the lowest at the surface of the specimen. The stress values at $z/w = 0.5$ are not plotted in the above two figures as they are almost the same as the $z/w = 0$ values.

In contrast to the through-thickness tensile stress distribution, the von Mises stress values, which are shown in Figure 12, increase with z from the centre until $z/w = 0.75$, then decrease as the free surface is approached. This is due to the combined effect of both σ_{xx} and σ_{zz} decreasing as the surface is approached.

3.1.3 Plane Strain Constraint Factor (γ) for the Sharp Notch Specimen

In Figure 13 are plotted the through-width variation of the plane strain constraint factors, γ , as calculated by equation (5). The plane strain constraint factor of the sharp notch specimen is observed to remain almost constant at a value reasonably

close to Poisson's ratio, and then drops abruptly after a small increase at $z/w = 0.7$. Theoretically, γ equals Poisson's ratio for plane strain, and zero for plane stress. The observed distribution of γ in the sharp notch specimen suggests a strong plane strain effect in the centre of the specimen up to a z/w value of 0.5. This is followed by a region where plane stress effects increase in importance up to the surface of the specimen.

3.2 Blunt Notch Specimen

3.2.1 Comparison of Stress Concentration Factors of Blunt Notch

Table 2 shows the stress concentration factors and percentage differences for the blunt notch specimen. As in the sharp notch specimen, the finite element plane stress and plane strain values should be the same, and the difference between them is due to numerical approximations. The maximum difference between the two-dimensional classical and the finite element solutions is 3%.

TABLE 2
Stress Concentration Factors and Percentage Differences
for Blunt Notch Specimen

Blunt Notch	Stress Concentration Factor Solution (Eq. 4)	
	k	% Diff.
2D Classical Solution	4.46	0.00
FE Solution Plane Stress	4.38	-1.88
FE Solution Plane Strain	4.45	-0.29
FE Solution 3D, Centre of Specimen	4.60	3.04

3.2.2 Stress Distribution and Plane Strain Constraint for the Blunt Notch Specimen

For the blunt notch specimen the origin $r = 0$ is located at the notch root (see Figure 2). The stress distributions of the blunt notch specimen follow many of the trends of its sharp notch counterpart:

1. The plane stress and plane strain tensile stress solutions are identical, and are close to the 3D results at $z/w = 0$ (Figure 14).

2. The von Mises stress values are generally highest for plane stress, followed by the three-dimensional values, and finally plane strain (Figure 15). An exception is near the notch root, where the von Mises stress for the three-dimensional model at $z/w = 1$ is the lower bound, due to the low value of σ_{xx} .
3. Three-dimensional tensile stress values do not vary significantly from $z/w = 0$ to $z/w = 0.5$. However, unlike the sharp notch specimen, they drop less abruptly to their minima at the surface (Figure 16).
4. von Mises stress values increase gradually from $z/w = 0$ to $z/w = 0.75$, and then drop to their minima at the surface (Figure 17).

On the other hand, the two specimens differ in the following aspects:

1. Stress concentration is less prominent in the blunt notch specimen, with its stress concentration factor roughly one half of the sharp notch value.
2. The plane strain constraint factor for the blunt notch specimen (Figure 13) is almost exactly zero at the surface, indicating a strong plane stress effect. It then increases gradually through the width to a value that is considerably smaller than Poisson's ratio at the mid-plane. Thus, a two-dimensional stress state, which is neither plane stress nor plane strain, exists between $z/w = 0$ and $z/w = 0.5$ in the blunt notch specimen.

4.0 CONCLUSIONS AND RECOMMENDATIONS

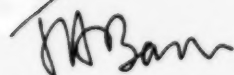
1. The tensile stress concentration factors from references 2 to 4 are in good agreement with the finite element solutions. All percentage differences are below or around 6%. The different approaches employed in these analyses yield average stress concentration factors of 7.93 and 4.47 for the sharp and blunt notch specimens, respectively.
2. The tensile stress values (σ_{xx}) from the plane stress and plane strain analyses are practically the same. They are close to the values obtained at the centre of the specimen from the three-dimensional models. Tensile stress values (σ_{xx}) obtained from the three-dimensional models indicate almost constant through-width profiles from $z/w = 0$ (the centre) to $z/w = 0.5$. This is followed by a decrease in values towards the free surface.
3. The von Mises equivalent stress values are generally highest for plane stress, and lowest for plane strain. The values obtained at different locations from the three-dimensional models are intermediate between the two. An exception is near the notch root, where the von Mises stress for the three-dimensional model at the free surface is the lower bound, due to the low value of tensile stress. The von Mises equivalent stress values do not indicate a definite trend as was the case for the tensile stress. Nevertheless, there is always an increase in values from the centre of the specimen, reaching a peak, followed by a decrease as one moves to the surface. This concurs with the general observation that more plastic yielding occurs near the surface than at the centre of a tension specimen at fracture.

4. The plane strain constraint factor (γ) of the sharp notch specimen indicates a plane strain state in the centre portion of the specimen from $z/w = 0$ to $z/w = 0.5$. A state which is neither plane stress nor plane strain, but is nevertheless two-dimensional, exists in the interior of the blunt notched specimen. The plane strain constraint factor is much lower than Poisson's ratio in the centre region of this specimen.
5. A two-dimensional analysis can suffice for the determination of the tensile stress σ_{xx} in the centre portion of either specimen ($z/w = 0$ to $z/w = 0.5$). Due to the definition of the von Mises stress, a three-dimensional analysis must be performed if von Mises values are desired.
6. The current analyses employed isotropic elastic material properties. Since the Zr-Nb pressure tube material is orthotropic, further analyses should be performed by using orthotropic elastic properties to assess the effect of material anisotropy on the stress fields in the notched tension specimens. Furthermore, plastic effects can be investigated by elastic-plastic finite element analyses.

5.0 ACKNOWLEDGEMENTS

The financial support of F. Kwok by the Natural Sciences and Engineering Research Council of Canada and the Centre for Nuclear Engineering of the University of Toronto for the term of this project is gratefully acknowledged.

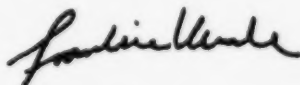
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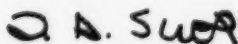
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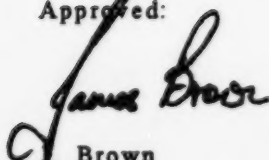


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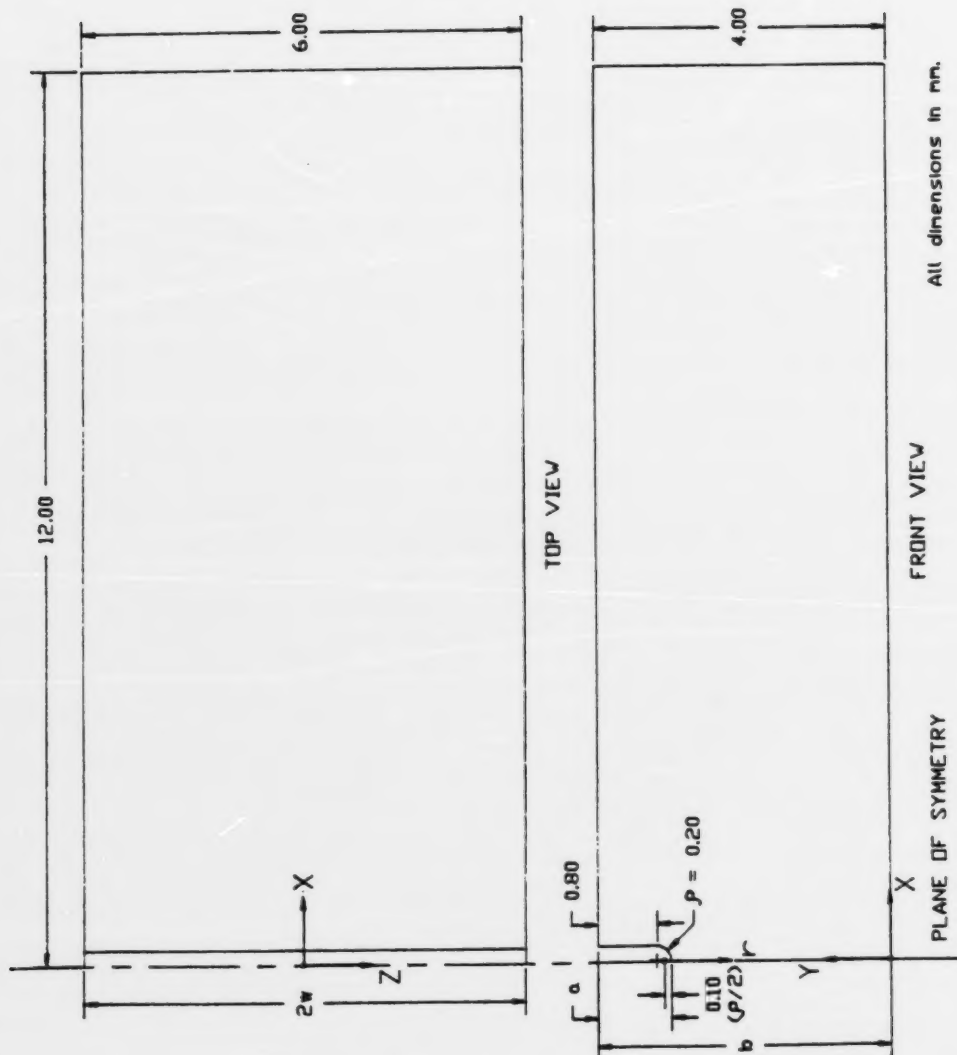


FIGURE 1
SHARP NOTCH TENSION SPECIMEN
ROOT RADIUS = 0.20 MM



FIGURE 2
BLUNT NOTCH TENSION SPECIMEN
ROOT RADIUS = 1.00 MM

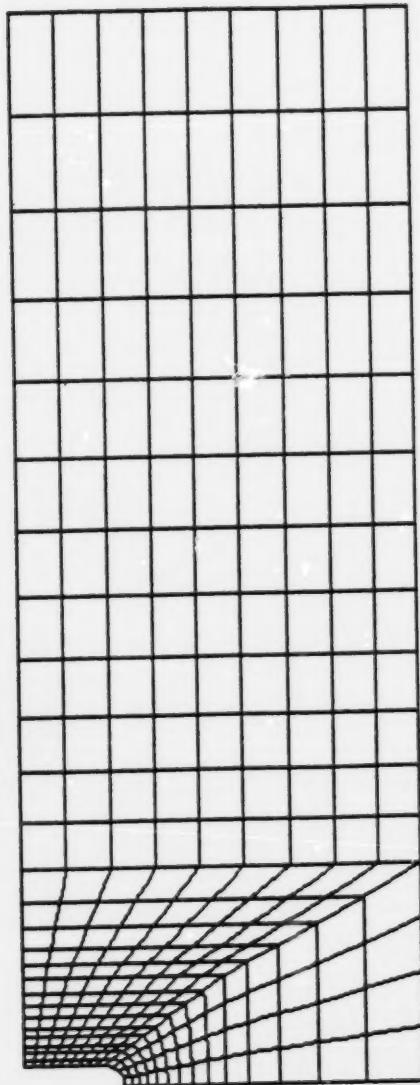


FIGURE 3
2D FINITE ELEMENT MESH OF
SHARP NOTCH TENSION SPECIMEN

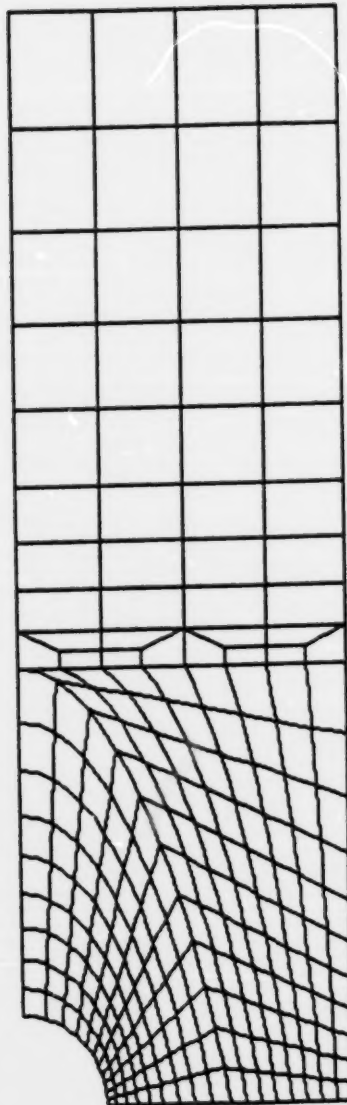


FIGURE 4
2D FINITE ELEMENT MESH OF
BLUNT NOTCH TENSION SPECIMEN (COARSE MESH)

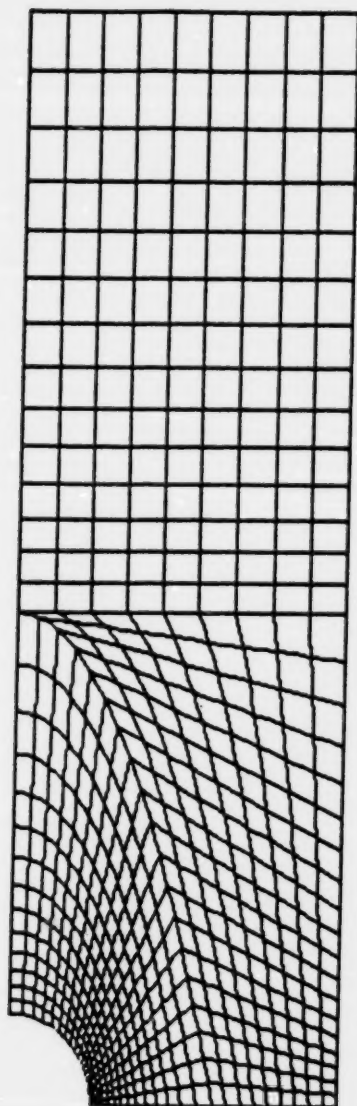


FIGURE 5
2D FINITE ELEMENT MESH OF
BLUNT NOTCH TENSION SPECIMEN (FINE MESH)

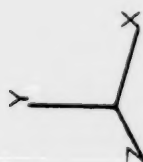
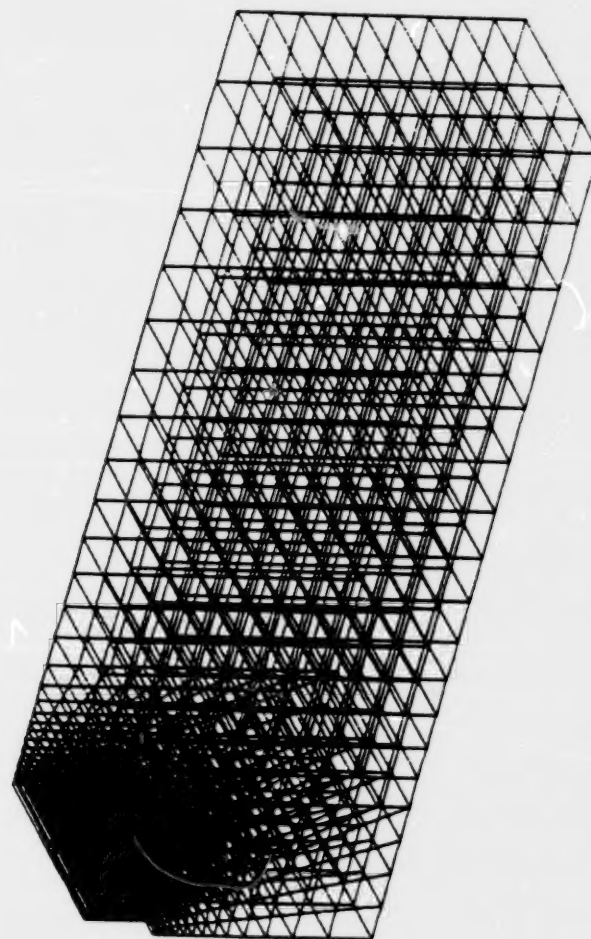


FIGURE 6
3D FINITE ELEMENT MESH OF
SHARP NOTCH TENSION SPECIMEN

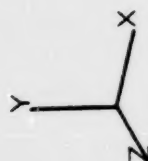
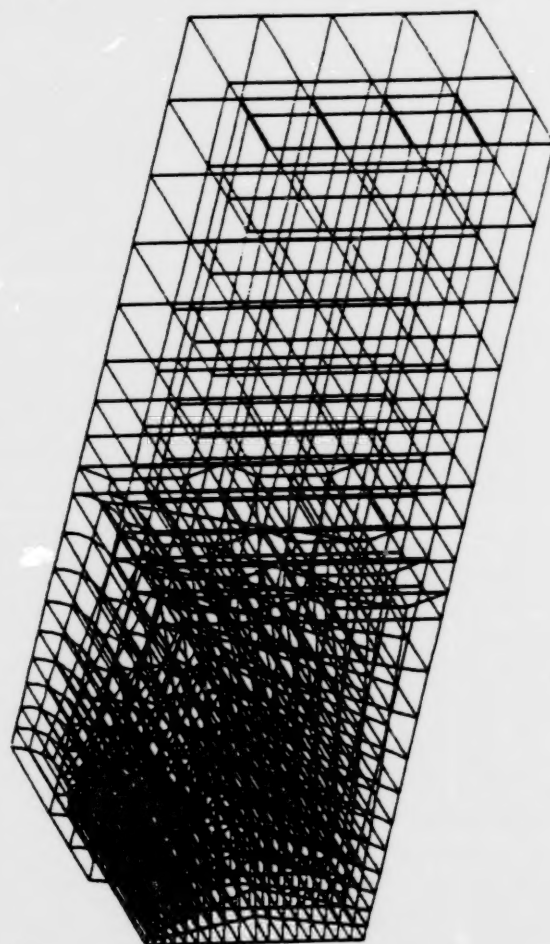


FIGURE 7
3D FINITE ELEMENT MESH OF
BLUNT NOTCH TENSION SPECIMEN

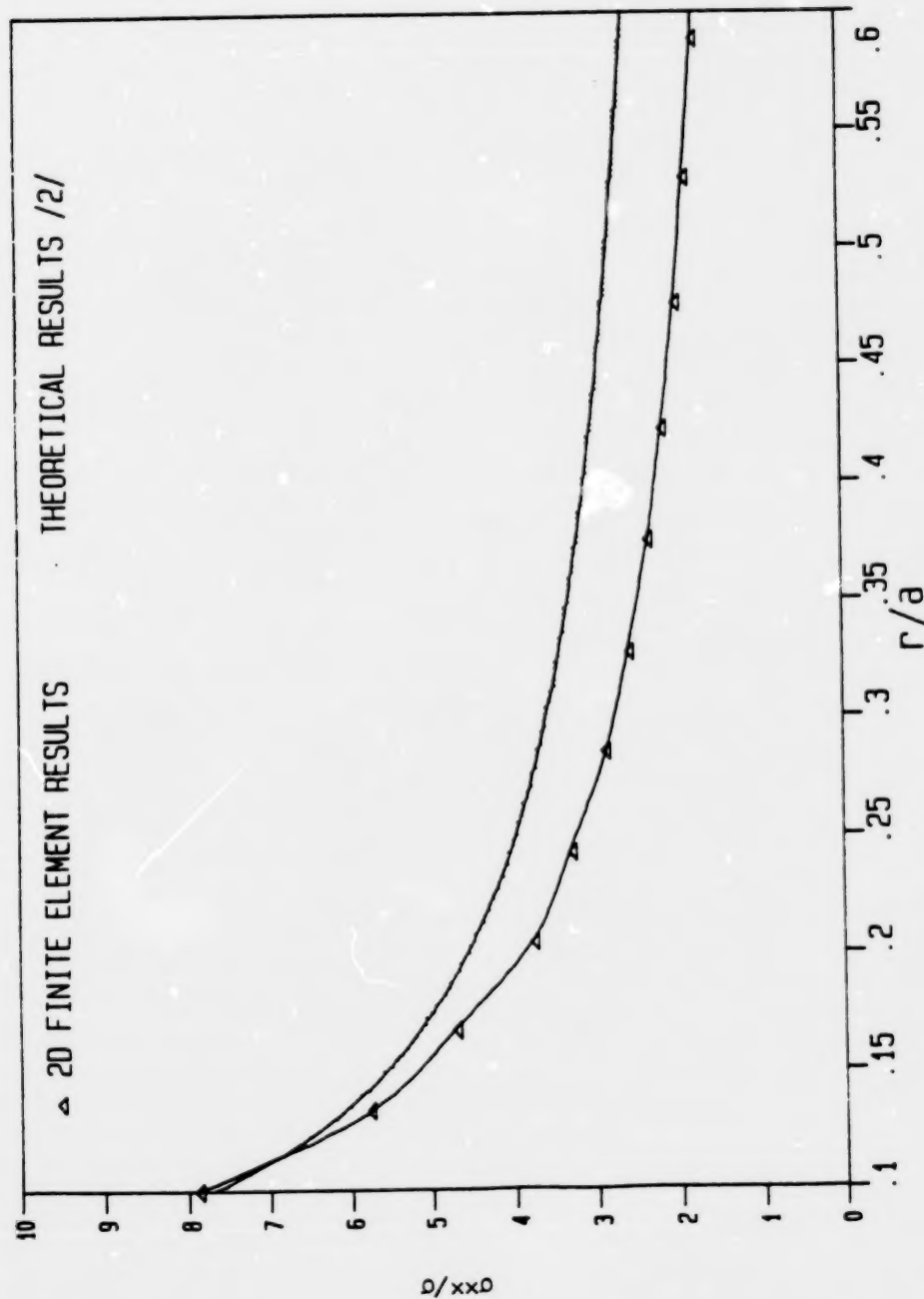


FIGURE 8

2D F.E. AND CLASSICAL RESULTS OF THE
THROUGH-THICKNESS VARIATION IN THE TENSILE STRESS
IN THE SHARP NOTCH TENSION SPECIMEN

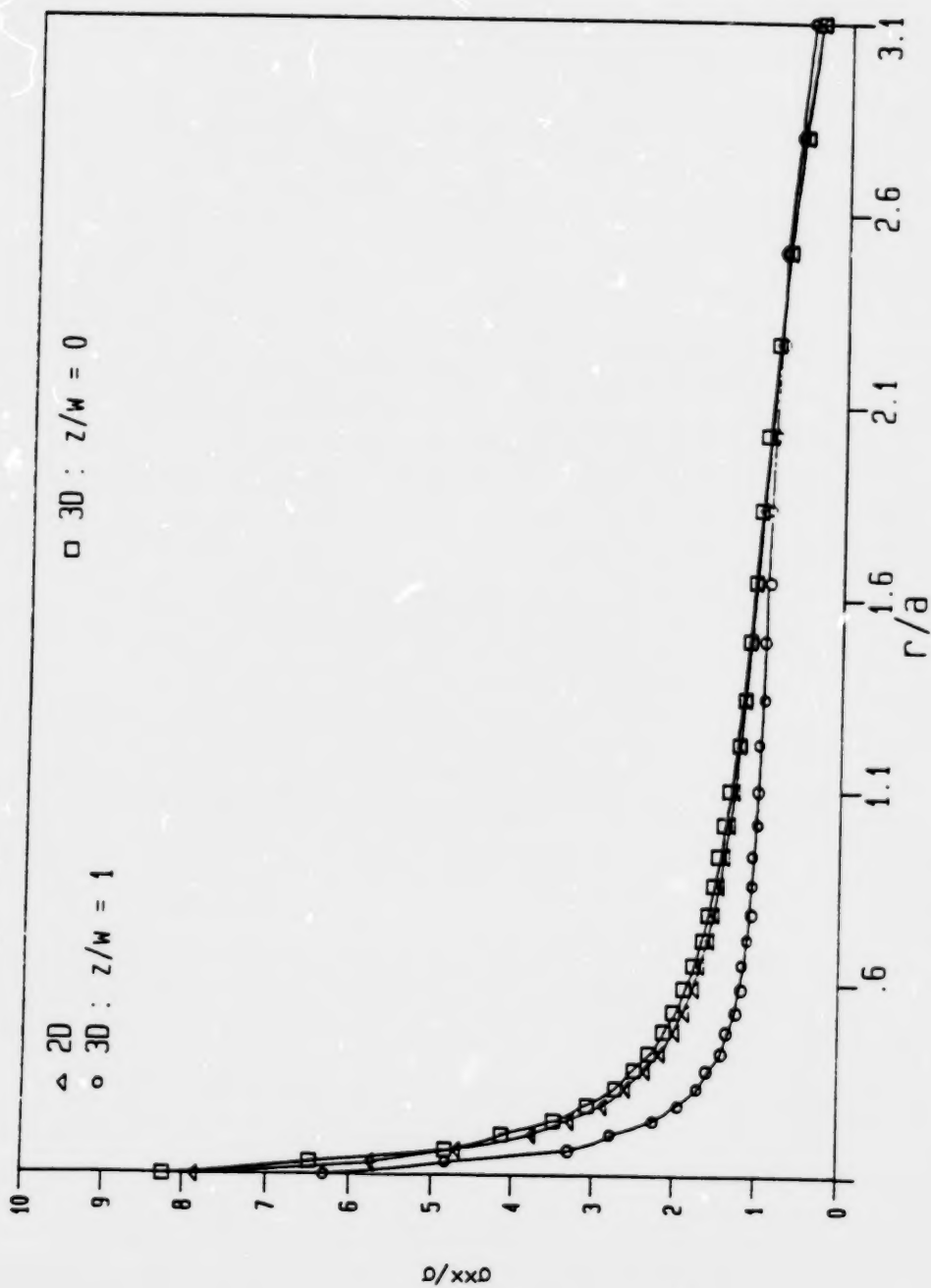


FIGURE 9
2D AND 3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE TENSILE STRESS IN THE
SHARP NOTCH TENSION SPECIMEN

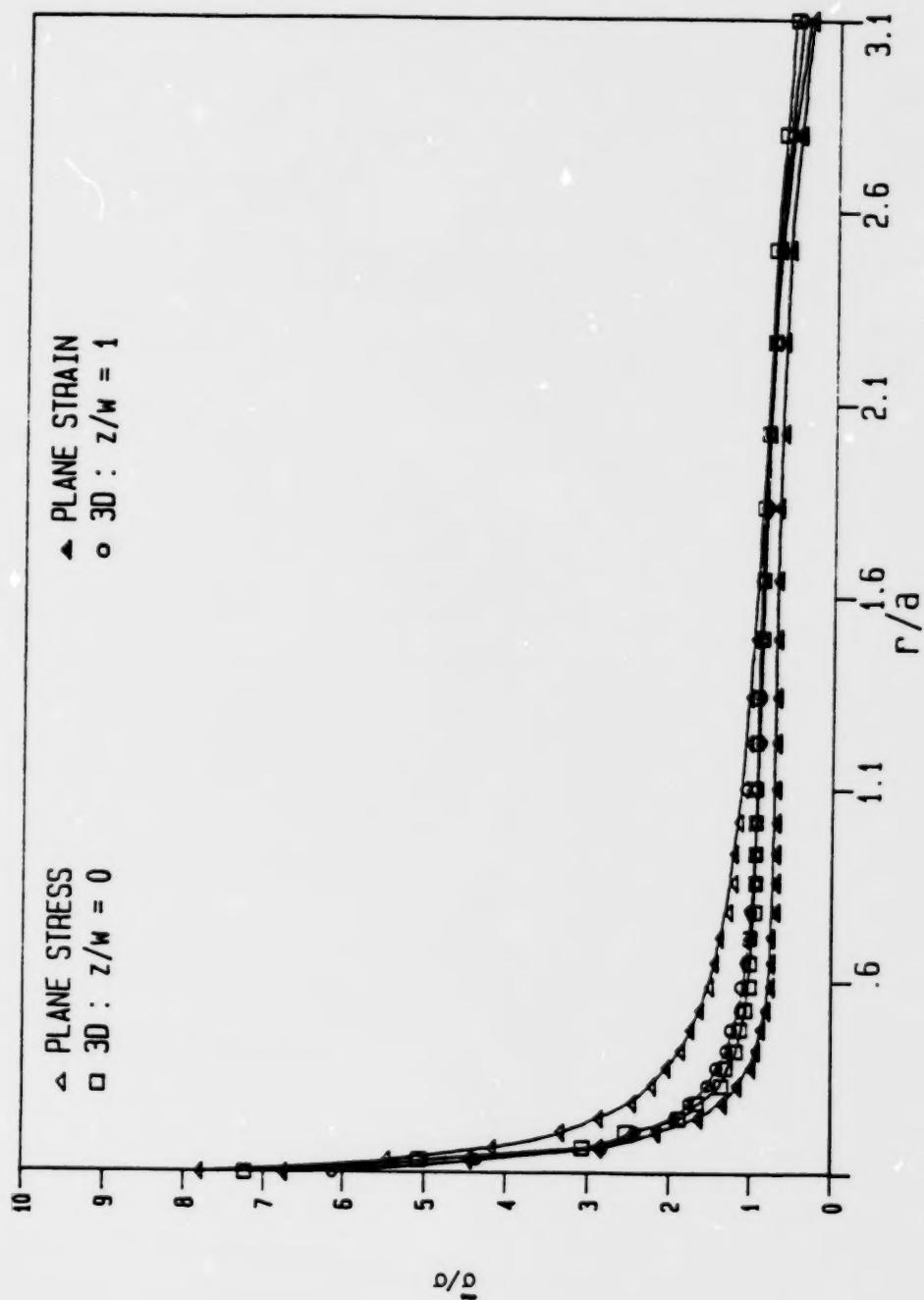


FIGURE 10
2D AND 3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE VON MISES STRESS IN THE
SHARP NOTCH TENSION SPECIMEN

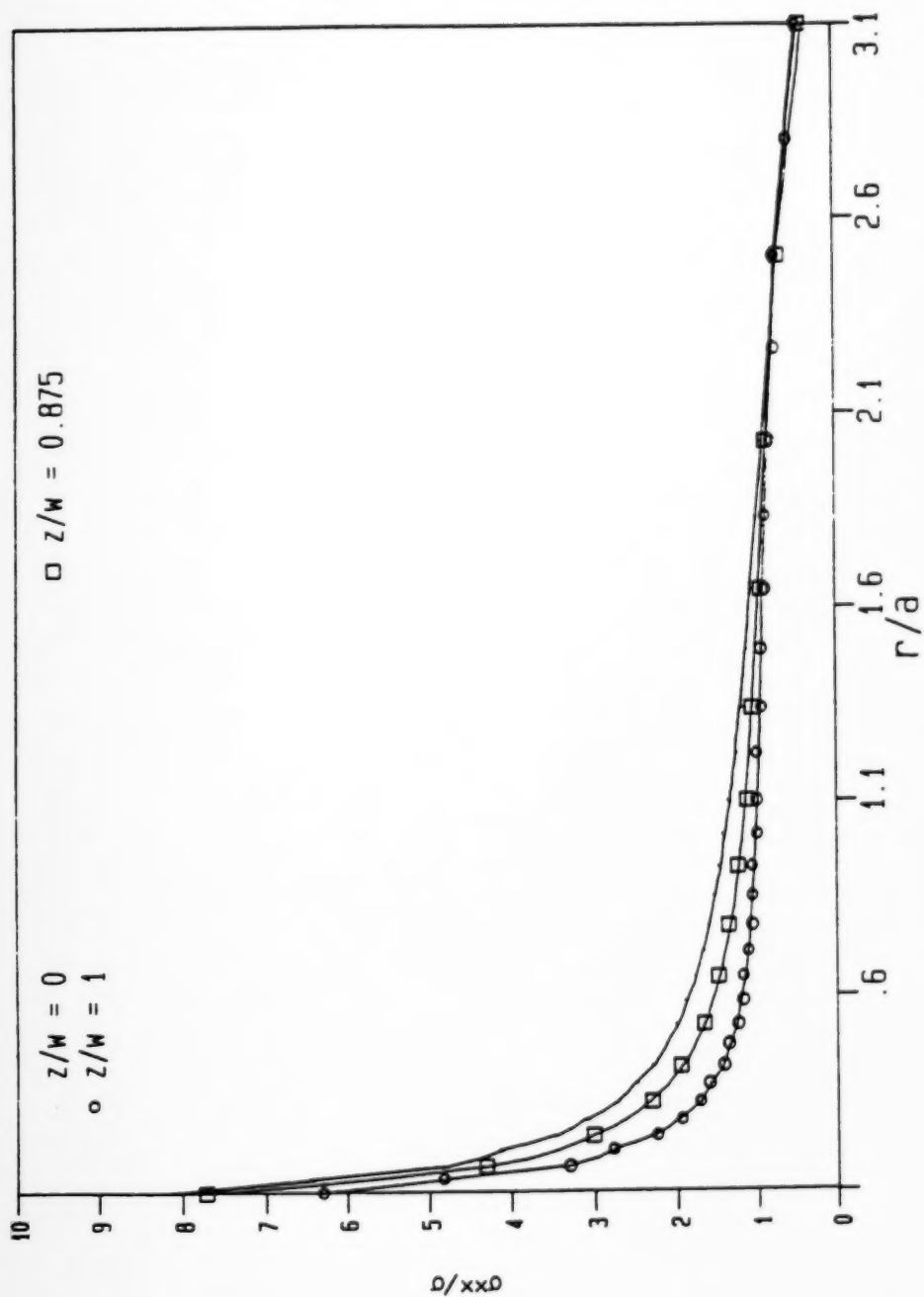


FIGURE 11
3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE TENSILE STRESS IN THE
SHARP NOTCH TENSION SPECIMEN

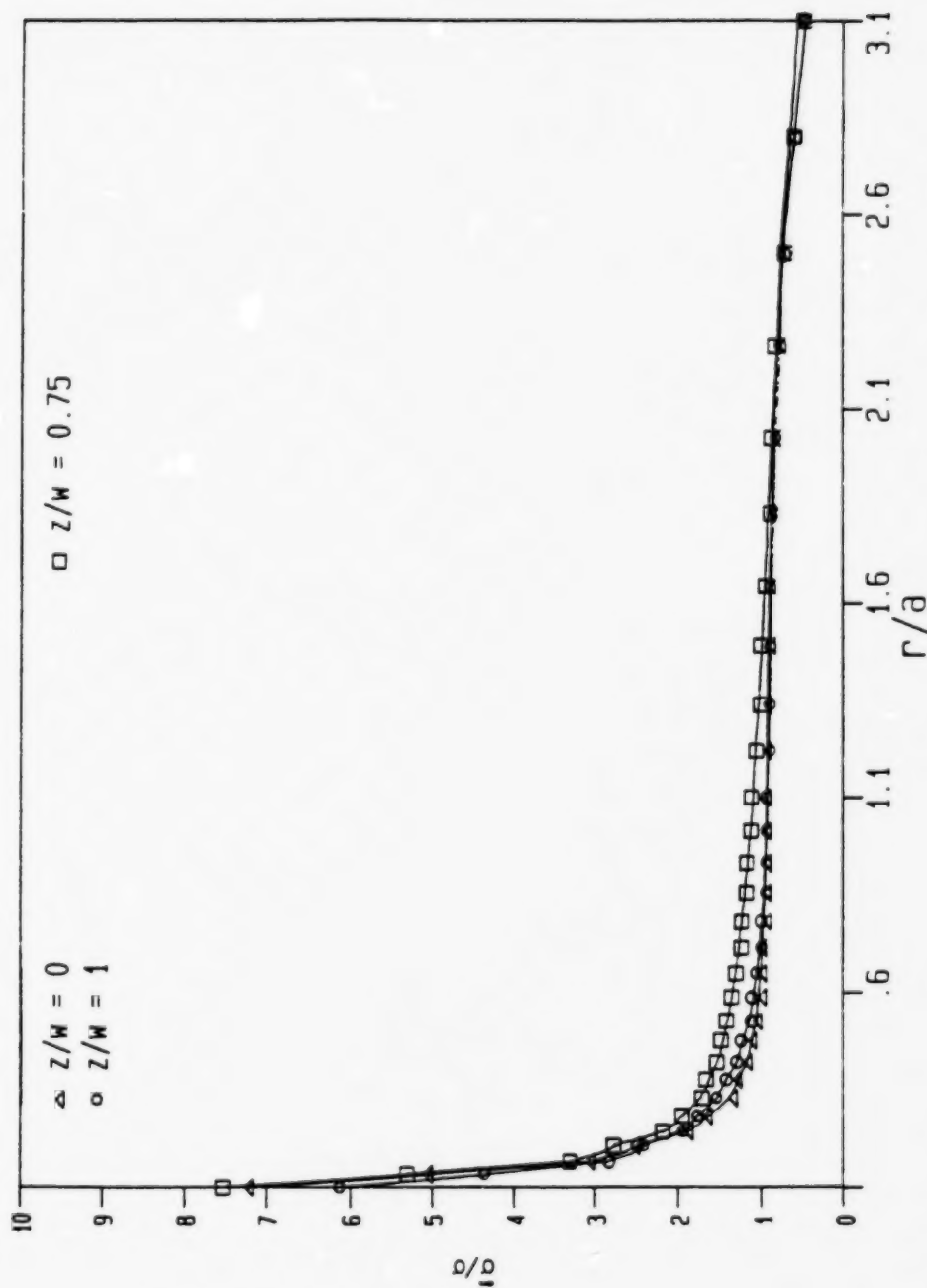


FIGURE 12
3D F.E. RESULTS OF THE THROUGH-THICKNESS VARIATION
IN THE VON MISES STRESS IN THE SHARP NOTCH
TENSION SPECIMEN

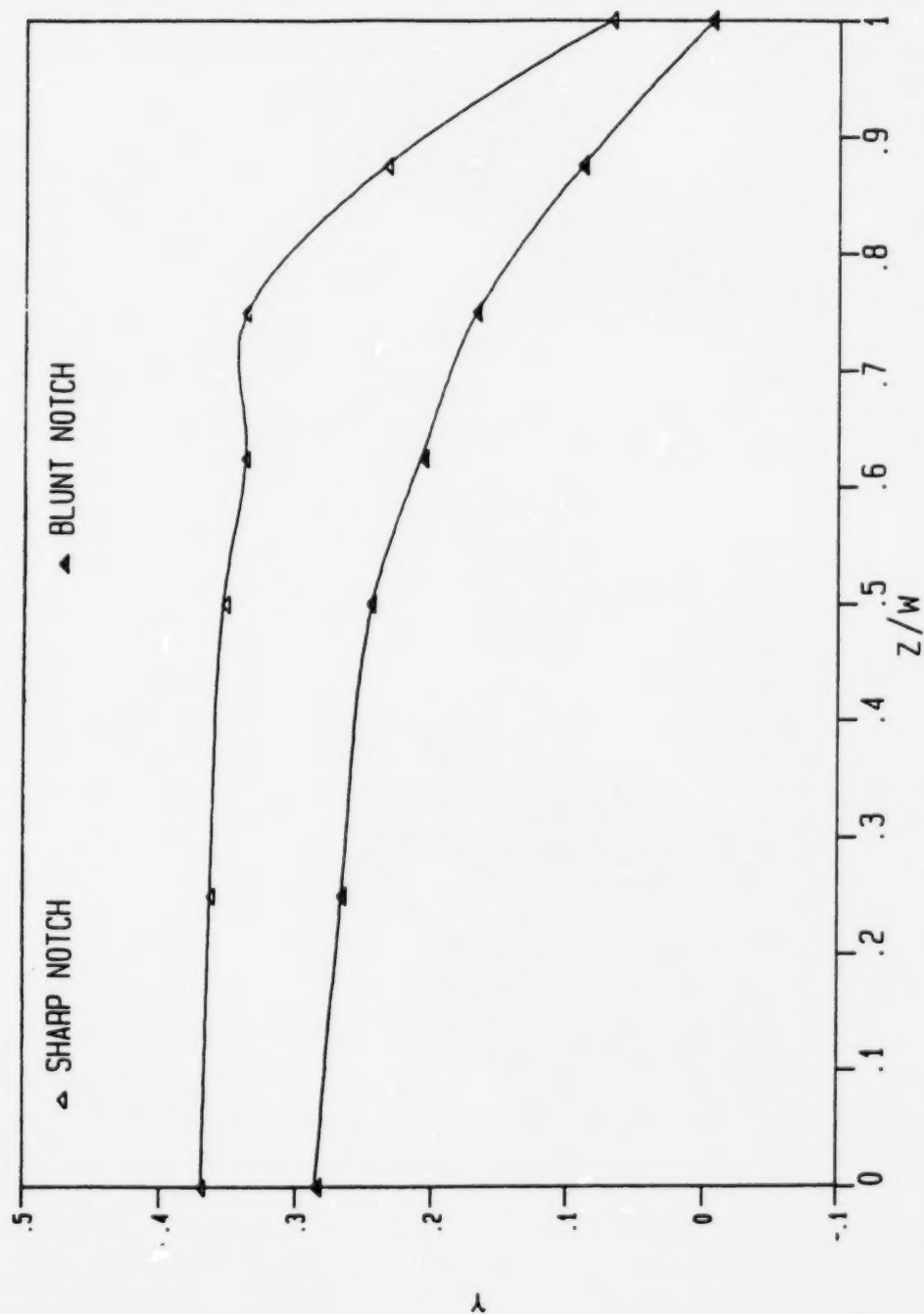


FIGURE 13
VARIATION IN THE PLANE STRAIN CONSTRAINT FACTOR AT
NOTCH TIP THROUGH THE SPECIMEN WIDTH

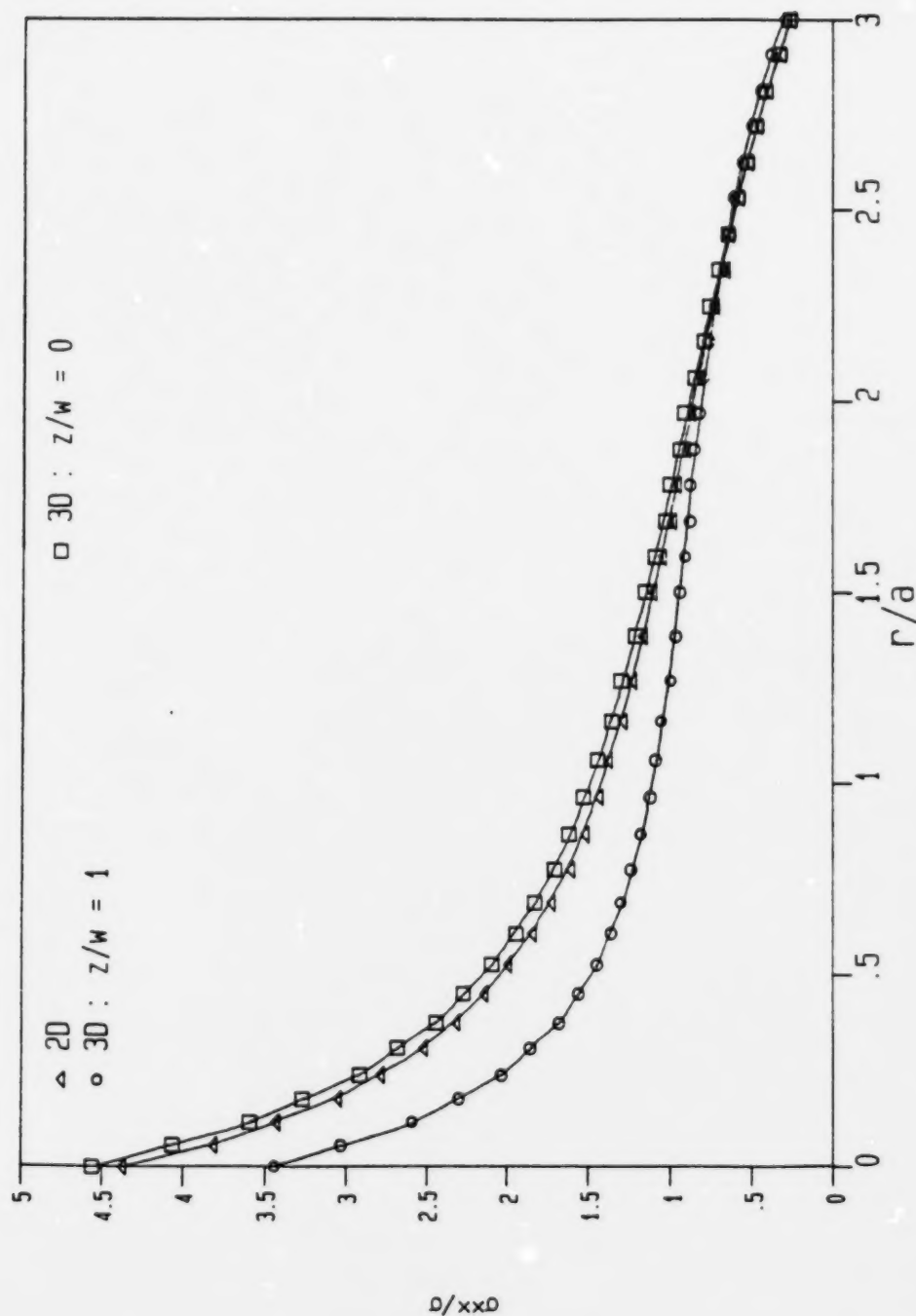


FIGURE 14
2D AND 3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE TENSILE STRESS IN THE
BLUNT NOTCH TENSION SPECIMEN

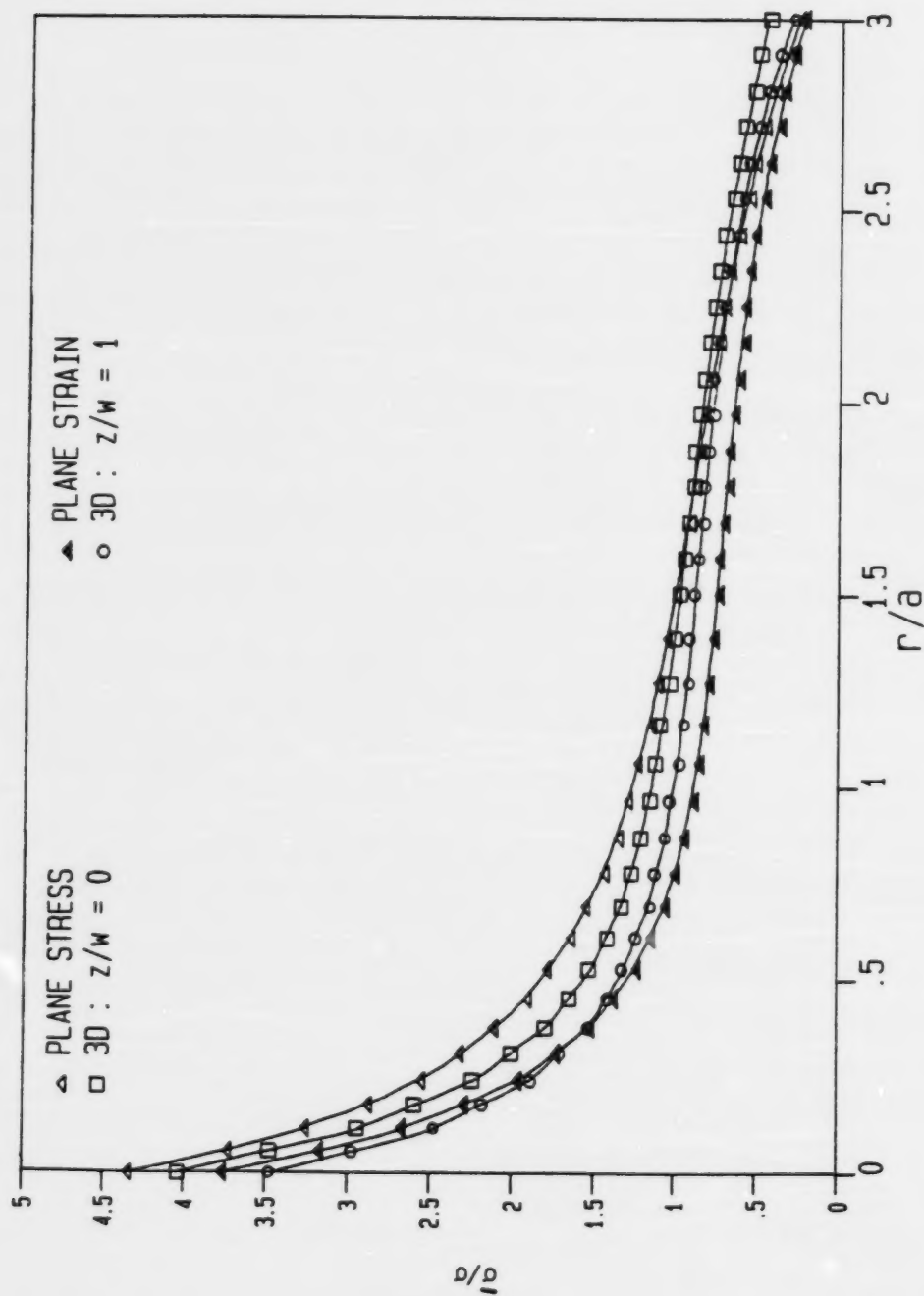


FIGURE 15

2D AND 3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE VON MISES STRESS
IN THE BLUNT NOTCH TENSION SPECIMEN

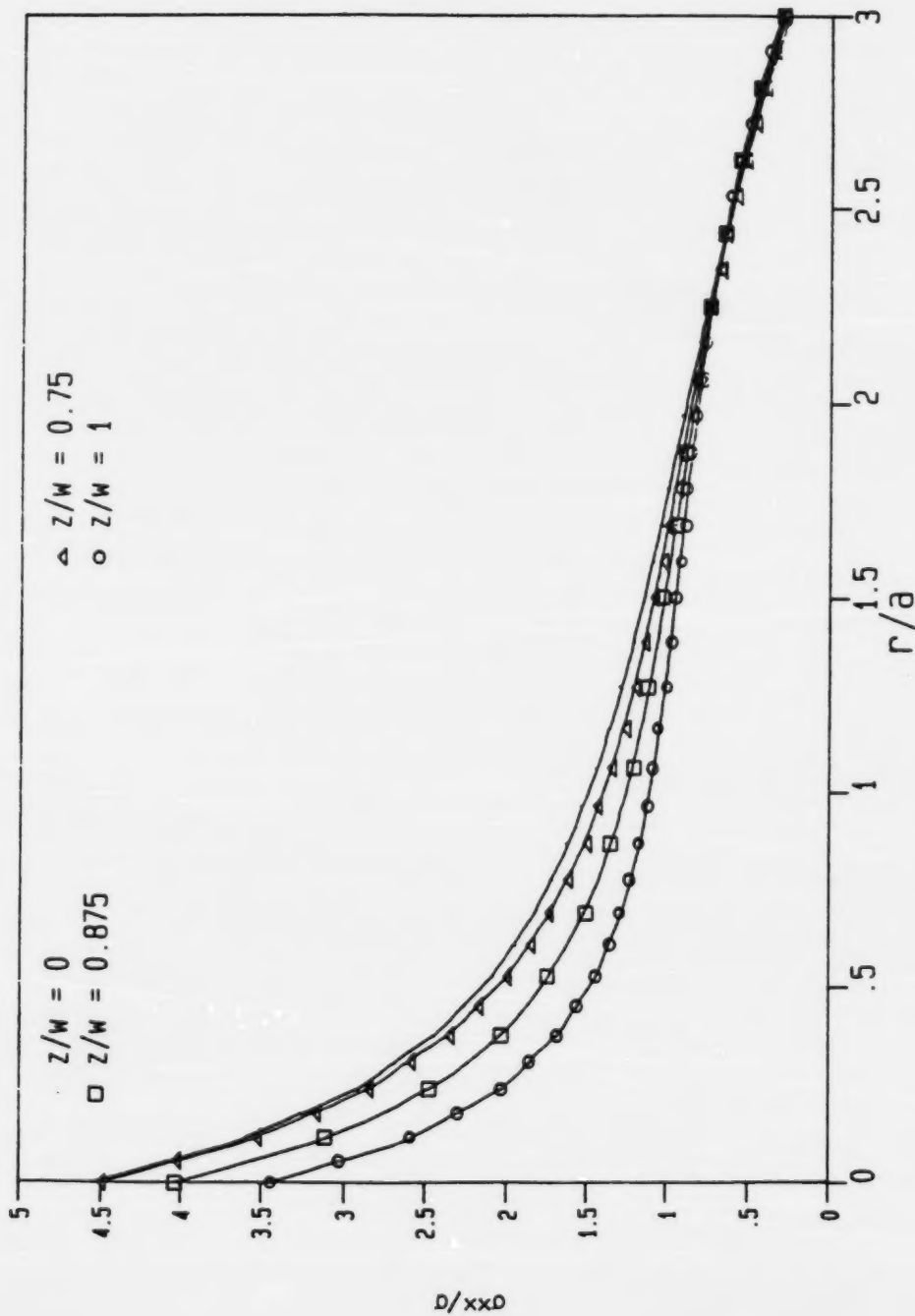


FIGURE 16
 3D F.E. RESULTS OF THE THROUGH-THICKNESS VARIATION
 IN THE TENSILE STRESS IN THE
 BLUNT NOTCH TENSION SPECIMEN

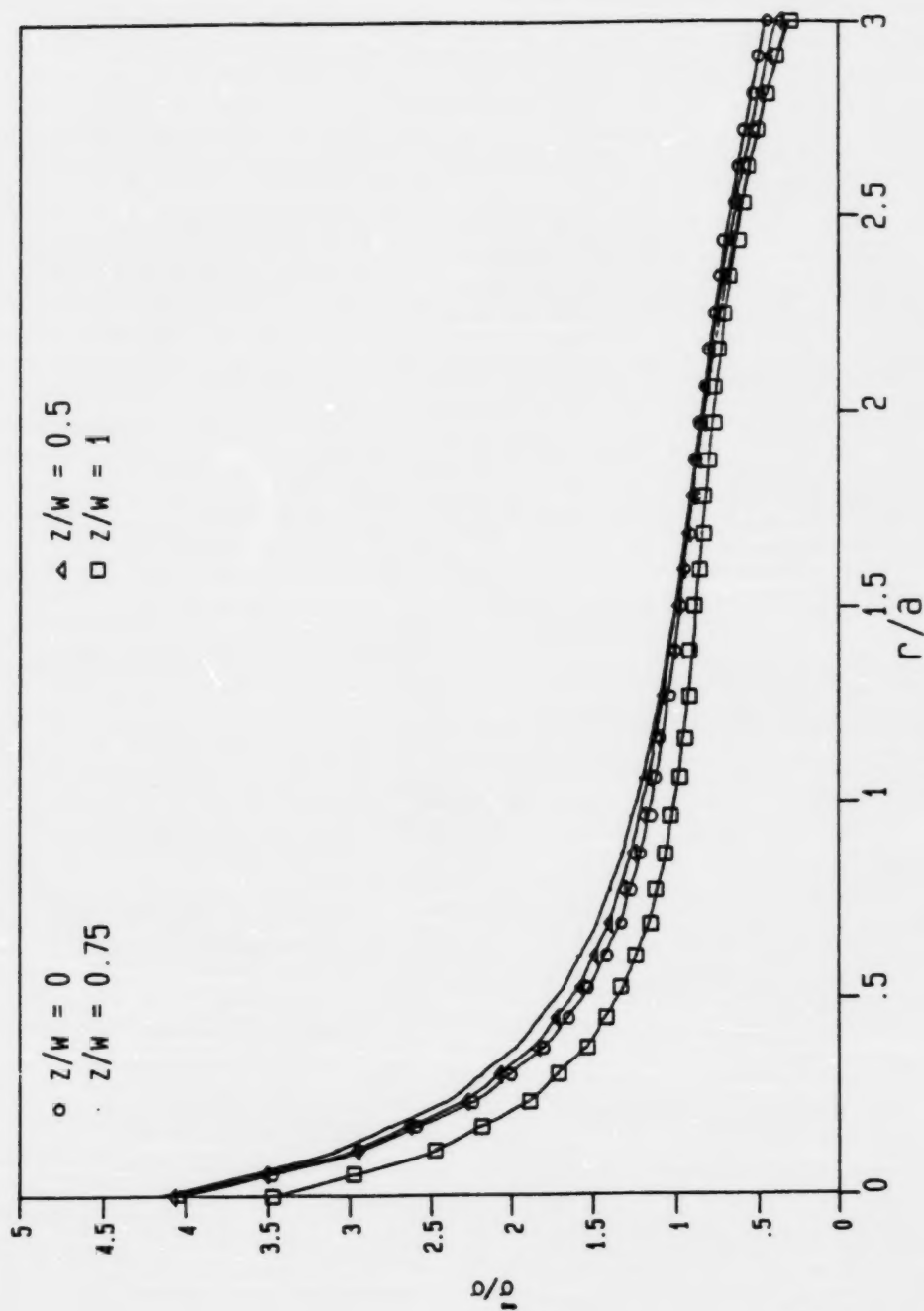


FIGURE 17
3D F.E. RESULTS OF THE THROUGH-THICKNESS
VARIATION IN THE VON MISES STRESS IN THE
BLUNT NOTCH TENSION SPECIMEN

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